Discrete Random Structures
MATH 4340/5340
Fall 2017

LECTURES: MWF 10:35–11:25, LSc 244. Web presence: on BrightSpace. No labs or tutorials.

INSTRUCTOR: Dr. Jeannette Janssen.

- Office: Chase building, room 315.
- Office hours: Tuesday 4-5.30pm, or by appointment.
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Course Description from Calendar: This course will cover basics of probability and stochastic processes, and then focus on areas where probability and combinatorics interact. Topics include: probabilistic method, stochastic graph models for complex networks, probabilistic algorithms. Probabilistic techniques include: expectation and concentration of random variables, stochastic processes, conditional expectation, Markov chains, martingales, branching processes.

Course Prerequisites: MATH4330, MATH 2113 or permission from instructor

Course Text: Class notes will be posted. I recommend that you get the following text:

- Probability and Random Processes, Grimmett and Stirzaker, Oxford University Press. In Killam library.

For additional reading:

- Mitzenmacher and Upfal, Probability and Computing.
- Alon, Erdös and Spencer, The Probabilistic Method.

Other relevant policies A document containing Dalhousie's policies on plagiarism, accessibility, and a number of other important issues has been posted to Brightspace.

Evaluation:

MATH 5340	Assignments:	50%
	Class presentations	25%
	Project :	25%
MATH 4340	Assignments:	50%
	Midterm exam	25%
	Final exam :	25%

Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale:

A+ (90-100)	B+ (77-79)	C+ (65-69)	D (50-54)
A (85-89)	B (73-76)	C (60-64)	F (¡50)
A- (80-84)	B- (70-72)	C- (55-59)	

Assignments. The evaluation of this course is based in large part on the assignments. The assignments enable the students to apply the probabilistic techniques learned in class. To ensure that the students reach a satisfactory level of competency, there will be a 2-step marking process. In the first round, problems are marked Pass or Fail. Only problems with a Pass are given points. Problems that Fail must be handed in again the next week. I encourage students to come see me for extra advice. Problems submitted in the second round can receive at most 85% of the maximum amount of points. Problems that receive a Pass may be handed in again for an improved grade. Assignments must be done individually; please read the university plagiarism policy, that can be found on the course Web page. Any material consulted must be properly credited; this includes web pages, or personal communication. For students taking this class as a graduate class (MATH 5340), there will be some separate assignment problems.

Exams (4340 only). For MATH 4340, there will be two take-home exams, one given at the end of October, the other in the last week of class.

In-class presentations (5340 only). For MATH 5340, students will be assigned readings of the course material, and they will have to present some of this material in class. All students will be expected to be able to answer questions on the assigned readings.

Term project (5340 only). In addition, MATH 5340 students will be expected to do a term project. This consists of reading and digesting a scientific paper which uses the probabilistic techniques taught in class. Students will give a short (20 minute) presentation on their topic, and produce a report of 10–15 pages.

Topics:

(1) Discrete probability spaces. Definition, events, independence, conditional probability. Applications and examples: the random graph $G(n, p)$, the probabilistic method applied to Ramsey numbers and expanding graphs, the Lovasz Local lemma and applications to graph colouring, random regular graphs.

(2) Discrete random variables, expectation, probability distributions (binomial, Poisson), indicator variables. Applications and examples: reliability of networks, the probabilistic method applied to dominating sets and Hamiltonian paths in graphs. Coupon collector problem. A randomized binary plane partition algorithm.

(3) Concentration results. Variance, Markov and Chebyshev inequalities, Chernoff bound. Applications and examples: global wiring in gate arrays, threshold functions for properties of the random graph.

(4) Poisson point processes. Applications and examples: Random geometric graphs.

(5) Generating functions and branching processes. Applications and examples: (linear) random walks, diameter in random graph.

(6) Markov chains. Markov property, stationary distribution, transition matrix. Applications and examples: random walks on graphs (hitting time, cover time) Markov Chain Monte Carlo (MCMC), applied to counting graph colourings, volume estimation.

(7) Martingales. Filter of a probability space. Conditional expectation. Azuma-Hoefding inequality. Sub- and supermartingales. Applications and examples: bin packing, Hamming distance, generative graph models for large networks.

Learning Outcomes:

- To learn the fundamental concepts of discrete probability: formal definition of probability space, random variable, expectation, conditional expectation, variance.
- To become familiar with basic stochastic processes, namely Markov processes, Poisson point processes, and branching processes.
- To learn *the probabilistic method*, i.e. to be able to apply basic probability concepts in order to solve combinatorial problems.
- To be familiar with the basic results on the Erdös-Renyi random graph model $G(n, p)$.
- (MATH 5340) To be able to read and understand a scientific paper where discrete probabilistic methods are used.
- To be able to analyze probabilistic algorithms.
- To understand and be able to use the basic terminology and notation used in extremal and probabilistic results, such as BigOh, smallOh, *with high probability*, *concentration*.